Reliable Adaptive Recoding for Batched Network Coding with Burst-Noise Channels

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***Abstract*—Network coding provides an efficient approach for multi-hop communication. The message from the source node is encoded into batches of coded packets, and intermediate nodes perform recoding steps to transmit the message to the destination node. Adaptive recoding optimizes the number of recoded packets per batch to enhance network throughput, accounting for fluctuations in packet loss. In this paper, we propose an adaptive recoding scheme in burst-noise channels with unknown channel parameters. We first provide uncertainty quantification for channel parameters using historical data and build a confidence set to cover true channel parameters with high probability. Next, we obtain the optimal recoding policy by solving a robust Markov decision process (MDP) problem, where uncertain parameters belong to the confidence set. The objective of the robust MDP is to optimize the worst-case reward function by considering all possible problem parameters from the confidence set. Experimental results demonstrate that our proposed recoding strategy significantly enhances network communication throughput with burst-noise channels.**

I. INTRODUCTION

Large-scale network communication often traverses multiple communication channels, each of which can introduce errors, particularly in scenarios like 5G IAB and IoT [1]. Although the traditional *decode-and-forward* scheme [2] achieves the *min-cut* communication capacity of the network, it incurs high computational and storage costs at intermediate nodes. Instead, batched network coding has emerged as a computationally and storage-efficient solution for multi-hop communication [3–6], without depending on feedback or information of the network topology. Some existing batched network codes, such as batched sparse (BATS) codes [3, 7], are known to achieve communication capacity with negligible gaps and are suitable to be deployed in extreme communication environments such as deep space [8, 9] and underwater [10–12].

A batched network code first encodes the message at the source node into batches of coded packets. Intermediate network nodes then perform re-encoding, or simply called *recoding*, to generate further recoded packets based on received packets. It is worth mentioning that the generation of recoded packets is restricted within the same batch. All received packets are decoded jointly at the destination node. The simplest recoding scheme, called *baseline recoding*, generates the same number of recoded packets regardless of the information contained in the received batches, but its network throughput may not be optimal in general [13]. Adaptive recoding is later proposed to determine the optimal number of recoded packets per batch. By adapting to fluctuations in the number of received packets, adaptive recording enhances communication efficiency [14–21]. However, this technique relies on precise knowledge of each communication link, which is difficult to obtain in practice due to observational errors or precision limitations.

In this paper, we focus on the design of adaptive recoding for communication with a special burst-noise channel, called the Gilbert-Elliott (GE) channels in literature, and consider the scenario where channel parameters are unavailable. Such a channel model has *good* and *bad* states, representing high and low transmission successful rates, respectively. Unlike the classical independent packet loss channel that assumes each transmission has *independent* transmission successful rate, the burst-noise channel is a more suitable choice to model or approximate the behavior of practical wireless communication systems [22–24]. Our contributions are as follows:

- We first reformulate the adaptive recoding as a finite-horizon Markov decision process problem, where the objective is to seek the optimal recoding policy at each hop such that the reward function balancing the throughput at the destination node and the processing complexity at all communication links is optimized. In particular, the transition dynamics of the MDP model depend on the parameters of burst-noise channels, which may not be reliably estimated.
- To tackle the challenge of estimating the parameters of burst-noise channels, we investigate the uncertainty quantification problem regarding those channel parameters. We first provide their point estimation using expectation–maximization (EM) algorithm based on a single trajectory of historical observations indicating whether the transmitted packet is successfully received. Based on multiple trajectories of observations, we build an elliptic confidence set that covers the true channel parameters with high probability. Such coverage is guaranteed asymptotically regard-

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ing the sample size and negligible EM optimization error.

- Finally, we propose solving the adaptive recoding problem by seeking the worst-case transition dynamics among an ambiguity set of the MDP model so that the worst-case MDP risk function is minimized. The ambiguity set is constructed by considering the parametric form of the transition dynamics in terms of the parameters from the burst-noise channel, whereas those parameters are assumed to belong to the constructed confidence set.
- Numerical simulations show that the proposed recoding strategy can improve network throughput by 119.12% compared with the baseline that assumes an independent loss channel model.

Notations. For integers $m \leq n$, define $[m:n] := \{m,m+1,...,n\}$. We write $[0:n]$ as $[n]$ for simplicity. We assume coding are performed in a finite field of size q. We use $\mathbb{P}_{\otimes}$ to indicate that the probability is evaluated with respect to the sampling distribution.

II. Problem Setup

A batched network code for a line network of length L, with nodes $0,1,...,L$, is formed by the following operations:

- The source node first encodes the input message as a number of batches, each of which has M packets. For each batch, recoding is performed based on the M packets to generate N_1 recoded packets using random linear combination (RLC) over field size q, which are then transmitted to the next node.
- For each batch of packets and for $\ell = 1,\dots,L-1$, the intermediate network node ℓ performs recoding to generate N_ℓ packets based on $N_{\ell-1}$ received packets, which are then transmitted to node $\ell+1$.
- The destination node L performs decoding based on all received packets over all batches.

For intermediate node $\ell \in [1:L-1]$, one can determine the number of recoded packets N_ℓ based on the information of received batches at this node. The number of recoded packets N_1 at the source node can be determined based on the whole network topology. Such a methodology is called *adaptive recoding* in literature [15, 25]. Based on this observation, we formulate the above problem as a Markov decision process [26], which consists of the following components:

a) Stage: The index of node, denoted as $\ell \in [L]$.

b) State: The rank of the received batch, denoted as $s_\ell \in [M]$, where M is the batch size at the source node.

c) Action: The number of recoded packets sent to the outcoming link, denoted as N_ℓ. Denote the policy that generates the action N_ℓ based on state s_ℓ by $\pi_\ell(\cdot|s_\ell)$. A large value of N_ℓ spends too much resources but does not improve network throughput very much. Therefore, we assume the action space is bounded such that $N_\ell \in [N_{\max}]$ for some positive integer $N_{\max} \in \mathbb{N}_+$.

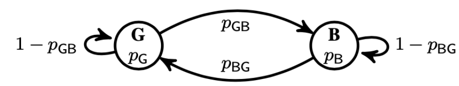

Fig. 1. Illustration of Gilbert-Elliott channel model

d) Reward Function: At stage $\ell \in [L-1]$, the reward $r_\ell(s_\ell, N_\ell) = -\eta \cdot N_\ell$, where $\eta > 0$ is some fixed parameter. At the final stage L, the reward $r_L(s_L) = s_L$.

e) Transition Dynamics: Given a current state s_ℓ and the action N_ℓ for $\ell \in [L-1]$, the next state $s_{\ell+1}$ follows a special rank distribution $P_\ell(\cdot|s_\ell, N_\ell)$. Let $P := \{P_\ell(s'|s, N)\}_{s,s' \in [M], N \in [N_{\max}], \ell \in [L-1]}$ denote the set of transition probability matrices, called the transition dynamics of the MDP. When considering the *batch-wise packet loss model* [27], i.e., for $\ell \in [L-1]$, the probability that node ℓ transmits s packets while node $\ell+1$ receives s' packets is given by $q_\ell(s' \mid s)$, as derived in [3]. Then, the following governs the transition dynamics of the system:

$$P_\ell(s|s_\ell,N_\ell) = \begin{cases} 0, & \text{if } s_\ell < s, \\ \sum_{k=s}^{N_\ell} q_\ell(k|N_\ell)\zeta_s^{s_\ell,k}, & \text{if } s_\ell \geq s. \end{cases} \quad (1)$$

Here $\zeta_j^{i,k}$ is the probability that $i \times k$ size matrix with independent entries uniformly distributed over the field of size q has rank j, which can be expressed in a closed-form as in [3, Eq. (2.4)]. For an independent loss channel with error probability ϵ, it holds that

$$q_\ell(k|N_\ell) = \binom{N_\ell}{k}(1-\epsilon)^k \epsilon^{N_\ell - k}. \quad (2)$$

The independent packet loss channel is generally too simple to approximate the practical communication channel. Instead, we focus on the Gilbert-Elliot channel model with parameters $(p_G, p_B, p_{GB}, p_{BG})$. For states good ($G$) and bad ($B$), the probability that a transmitted packet is lost is p_G and p_B, respectively. The transition between states G and B can be represented with a two-state Markov chain with transition probabilities p_{GB},p_{BG}, respectively as illustrated in Fig. 1.

Proposition 1. Denote by $q(\cdot|N)$ the distribution of received packets provided that N packets are transmitted from the incoming node. Suppose the transmission channel per batch is the GE channel with parameters (p_G,p_B,p_{GB},p_{BG}), then the probability mass value $q(k|N) = g_{N,k} + b_{N,k}$, where the parameters $g_{N,k}, b_{N,k}$ can be computed recursively.

- $g_{0,0} = \frac{p_{BG}}{p_{GB}+p_{BG}}$, $\quad b_{0,0} = \frac{p_{GB}}{p_{GB}+p_{BG}}$;
- $g_{n+1,k} = (1-p_{GB})p_G g_{n,k} + (1-p_{GB})(1-p_G)g_{n,k-1}$ $+ p_{BG}p_B b_{n,k} + p_{BG}(1-p_B)b_{n,k-1}$;
- $b_{n+1,k} = p_{GB}p_G g_{n,k} + p_{GB}(1-p_G)g_{n,k-1}$ $+ (1-p_{BG})p_B b_{n,k} + (1-p_{BG})(1-p_B)b_{n,k-1}$;
- $g_{n,k} = 0, b_{n,k} = 0$ for undefined indices (n,k).

Given a policy $\pi := (\pi_0, \pi_1, \ldots, \pi_{L-1})$ and transition dynamics $P := \{P_\ell(s'|s,N)\}_{s,s',N,\ell}$ we define the expected total reward as:

$$R(\pi;P) = \mathbb{E}^{\pi,P}\Big[\sum_{\ell\in[L-1]} r_\ell(s_\ell,N_\ell) + r_L(s_L)\Big], \qquad (3)$$

where the expectation is taken with respect to the stochastic policy π and transition dynamics P conditioned on the initial state $s_0 = M$. Once parameters $\theta := (p_G, p_B, p_{GB}, p_{BG})$ are known, the optimal recoding policy π can be computed using value iteration algorithm [26].

In this paper, we focus on the scenarios where channel parameters are unknown. In the following, we establish confidence sets for these parameters at hop $\ell = 0,\ldots,L-1$, denoted as Ξ^ℓ. Then, we consider the robust MDP formulation, where we seek the optimal policy π to maximize the worst-case reward function as follows:

$$\inf_{P\in\mathcal{P}} R(\pi;P), \qquad (4)$$

where $\mathcal{P}$ denotes the set of transition probability matrics $\mathcal{P}_\ell$ for stage $\ell \in [L-1]$, i.e., $\{P_\ell(s'|s,N)\}_{s,s',N} \in \mathcal{P}_\ell$. To maintain computational traceability, we specify the ambiguity set $\mathcal{P}_\ell$ as a (s,a)-rectangular set [28] of the following form:

$$\mathcal{P}_\ell = \Big\{P_\ell : P_\ell(\cdot|s,N) \in \mathfrak{M}^\ell_{s,N}, \forall s,N\Big\}, \qquad (5)$$

with $\mathfrak{M}^\ell_{s,N}$ representing the uncertainty set containing the transition kernel $P_\ell(\cdot|s,N)$ of the form (1) with parameter $\theta \in \Xi^\ell$.

In the next section, we will specify the construction of the uncertainty set Ξ^ℓ. After that, one can optimize the objective function (4) using the *robust value iteration* [29] as follows: For $\ell = L$, $V_L(s_L) = r_L(s_L)$. For $\ell = L-1,\ldots,0$, we solve:

$$Q_\ell(s_\ell,N_\ell) = \sup_{P_\ell(\cdot|s_\ell,N_\ell)\in\mathfrak{M}^\ell_{s_\ell,N_\ell}} \Big\{ r_\ell(s_\ell,N_\ell) + \sum_{s_{\ell+1}} P_\ell(s_{\ell+1}|s_\ell,N_\ell) V_{\ell+1}(s_{\ell+1}) \Big\}, \qquad (6)$$

for all state-action pairs (s_ℓ,N_ℓ) and

$$V_\ell(s_\ell) = \min_{N_\ell} Q_\ell(s_\ell,N_\ell), \quad \forall s_\ell. \qquad (7)$$

The optimal reward of the robust MDP equals the value function $V_0(s_0 = M)$. Robust MDP is a useful model for sequential decision-making problems with parameter uncertainty [30]. The parameter uncertainty set is a key ingredient of any robust MDP model. A good uncertainty set should be flexible enough such that it contains true model parameters with a high confidence level. It should also not be too large to avoid overly conservative decisions. In literature, various uncertainty sets have been proposed, such as those based on moment statistics or probability divergences [28, 29, 31, 32]. We herein adopt these references to construct a moment-based uncertainty set for our robust MDP model. Furthermore, we consider its special applications in adaptive network coding and provide novel statistical guarantees and optimization algorithms.

III. Estimation of GB Channel Parameters

In this section, we provide uncertainty quantification regarding the parameters of a given GE channel $\theta := (p_G, p_B, p_{GB}, p_{BG})$ based on historical data. Denote by $\mathcal{D} = \{(X^i_j, Y^i_j)\}_{j\in[n],i\in[1:m]}$ the *ground truth* data consisting of m independent and identically distributed (i.i.d.) trajectories, with the i-th trajectory being $((X^i_0, Y^i_0), (X^i_1, Y^i_1), \ldots, (X^i_n, Y^i_n))$. Here X^i_j is a *latent* Boolean-valued variable, indicating whether the channel state is good ($X^i_j = 1$) or bad ($X^i_j = 0$), and Y^i_j is a Boolean-valued observation variable indicating whether the j-th packet from i-th trajectory is successfully transmitted or not. Assume only samples $\mathcal{D}_o = \{Y^i_j\}_{j\in[n],i\in[1:m]}$ are observed, and i.i.d. variables $X^i_0, i \in [1:m]$ follow the known initial distribution ν, based on which we will provide the uncertainty quantification of parameter θ. In practice, this information can be obtained by setting up a feedback between the transmitter and receiver in each communication hop.

a) Point Estimation: We first give a point estimation of θ using the expectation–maximization (EM) algorithm. We herein extend the work of [33], which studied the parameter estimation problem of binary symmetric channels, to the packet loss channels. Algorithm 1 summarizes the overall EM algorithm for estimating the channel parameter θ. Specifically, the posterior probability mass values $\phi_{k|n}$ and $\phi_{k:k+1|n}$ in Step 2 of Algorithm 1 can be obtained by standard techniques from decoding in hidden Markov models [34].

b) Uncertainty Quantification: If the obtained solution from EM algorithm is equal to the optimal solution $\widehat{\theta}$ from solving the maximum likelihood estimation, by [34, Chapter 12], it holds that $n^{1/2}(\widehat{\theta} - \theta^*)$ asymptotically converge to a normal distribution for some covariance matrix Σ that can be estimated from data. Since it is assumed that we have m i.i.d. trajectories $\{Y^i_j\}_{j\in[n],i\in[1:m]}$, we can obtain m estimators $\widehat{\theta}^{(1)},\ldots,\widehat{\theta}^{(m)}$ using Algorithm 1, and then construct a set estimation of θ as:

$$\Xi = \left\{\theta : (\theta - \overline{\theta})\widehat{\Sigma}^{-1}(\theta - \overline{\theta}) \le \frac{T^2_{4,m-4}(1-\alpha)}{m}\right\}. \qquad (8)$$

Here $\overline{\theta}, \widehat{\Sigma}$ are sample mean and sample covariance over $\widehat{\theta}^{(1)},\ldots,\widehat{\theta}^{(m)}$, respectively, and $T^2_{4,m-4}(1-\alpha)$ is the $(1-\alpha)$-critical value of F-distribution with parameters 4 and $m-4$ (see [35] for detailed discussion of F-distribution). Consequently, as $n,m \to \infty$, it holds that $\mathbb{P}_\otimes\{\theta \in \Xi\} = 1-\alpha$, i.e., Ξ is the asymptotic confidence set of the ground truth channel parameter θ. One can repeat this procedure to generate uncertainty set Ξ^ℓ at each hop $\ell \in [L-1]$ for solving (4).

Remark 1 (Optimization for Problem (6))**.** The most computationally challenging part of solving the robust MDP

Algorithm 1 Point Estimation of Channel Parameter

Require: Initial guess $\widehat{\theta}$, single trajectory $\{Y_j\}_{j\in[n]}$.

while not convergent **do**

For $k\in[n], i\in[1], j\in[1]$, compute posterior probability mass values

$$\phi_{k|n}(i) = \mathbb{P}_{\widehat{\theta}}(X_k = i \,|\, Y_{0:n}),$$
$$\phi_{k:k+1|n}(i,j) = \mathbb{P}_{\widehat{\theta}}((X_k, X_{k+1}) = (i,j) \,|\, Y_{0:n}).$$

Update $\widehat{\theta} := (\widehat{p}_B, \widehat{p}_G, \widehat{p}_{BG}, \widehat{p}_{GB})$ with

$$\widehat{p}_B = \frac{\sum_{k=0}^{n} \phi_{k|n}(0) Y_k}{\sum_{k=0}^{n} \phi_{k|n}(0)},$$
$$\widehat{p}_G = \frac{\sum_{k=0}^{n} \phi_{k|n}(1) Y_k}{\sum_{k=0}^{n} \phi_{k|n}(1)},$$
$$\widehat{p}_{BG} = \frac{\sum_{k=1}^{n} \phi_{k-1:k|n}(0,1)}{\sum_{k=1}^{n}\sum_{l=0}^{1} \phi_{k-1:k|n}(0,l)},$$
$$\widehat{p}_{GB} = \frac{\sum_{k=1}^{n} \phi_{k-1:k|n}(1,0)}{\sum_{k=1}^{n}\sum_{l=0}^{1} \phi_{k-1:k|n}(1,l)}.$$

end while

Return $\widehat{\theta}$

formulation is to estimate the worst-case transition probability in Problem (6), which amounts to solving the parametric finite-dimensional optimization problem

$$\sup_{\theta\in\Xi^\ell} \Big\{ r_\ell(s_\ell, N_\ell) + \sum_{s_{\ell+1}\in[s_\ell]} \sum_{k=s_{\ell+1}}^{N_\ell} q_\ell(k|N_\ell) \zeta_{s_{\ell+1}}^{s_\ell,k} V_{\ell+1}(s_{\ell+1}) \Big\}.$$

In the formulation above, the objective depends on the decision variable θ through the packet loss model $q_\ell(\cdot|N_\ell)$, whose detailed expression can be obtained from Proposition 1. Due to the non-concavity of the objective function, we apply the Iterative Fast Gradient Method (IFGM) proposed in [36] to approximately solve this problem. Herein, the idea is to iteratively optimize the first-order Taylor expansion of the objective function around the last iteration point.

IV. Numerical Study

We specify the following hyper-parameter in our numerical study: $L = 10$, $M = 20$, $\eta = 0.01$. Each communication link is an identical GB channel with parameter $(p_G, p_B, p_{GB}, p_{BG}) = (0.7, 0.3, 0.25, 0.8)$.

We first validate the performance of uncertainty quantification regarding the channel parameter. Figure 2 reports the confidence set estimation of these 4 parameters projected on each coordinate across different sample sizes n with $m = 20$ trajectories and confidence level $\alpha = 0.05$. From the plot, we observe the constructed confidence intervals guarantee the coverage of ground truth channel parameters with high probability, especially for large sample sizes. Besides, the width of confidence intervals becomes smaller as the sample size increases, which suggests our provided algorithm gives a sample-efficient uncertainty quantification regarding unknown parameters.

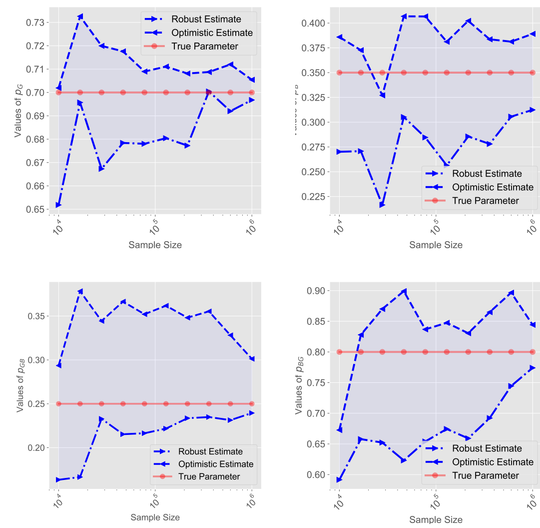

Fig. 2. Uncertainty quantification of channel parameters for different sample sizes n. Plots from left to right correspond to channel parameters p_G, p_B, p_{GB}, p_{BG}, respectively.

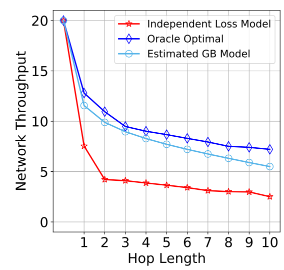

Fig. 3. Network throughput by various adaptive recoding approaches. Compared with the independent loss baseline model, our framework improves the throughput by 119.12%.

Next, we compare the performance of our adaptive recoding framework with two approaches: the baseline that mistakenly assumes the communication channel has an independent packet loss model and the *oracle optimal* method in which the channel model is assumed to be exactly known. These approaches optimize the MDP reward (3) with known transition dynamics, which can be solved using the standard value iteration algorithm [26]. From the plot in Figure 3, we realize that our proposed method significantly improves the throughput over using an independent packet loss model, and it has small sub-optimality gap in comparison with the oracle optimal.

V. Conclusion

We provided a robust MDP framework to design a reliable adaptive network recoding strategy under scenarios where channel parameters cannot be accurately obtained. An interesting research direction is to incorporate more sophisticated communication requirements into the adaptive recoding framework, such as multi-cast and multi-user communication with fairness constraints. Finally, for the networks with ultra-large state space, it is desirable to employ some low-complexity approximation algorithms such as [37] to reduce the complexity of robust value iteration algorithms.

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