

ISyE 3770 Assignment 5: Point Estimator

Due date: 11:59 PM, Friday, March 29, 2024.

Question 1 (MSE). Let X_1, X_2 be independent random variables with mean μ and variance σ^2 . Suppose we have two estimators of μ :

$$\hat{\Theta}_1 = \frac{X_1 + X_2}{2}$$

$$\hat{\Theta}_2 = \frac{X_1 + 3X_2}{4}.$$

- 1) Are both estimators unbiased? (8 points)
- 2) What is the variance of each estimator? (8 points)
- 3) What is the MSE of two estimators? (8 points)

Question 2 (MSE). Suppose $X \sim \text{Uni}(\theta, 3\theta)$ with $\theta > 0$. Let X_1, \dots, X_n be n i.i.d. random variables following the same distribution as X .

- 1) Prove that $\frac{\bar{X}}{2}$ is an unbiased estimator of θ . (8 points)
- 2) Calculate the MSE of $\frac{\bar{X}}{2}$ and \bar{X} . (8 points)

Question 3 (Maximum Likelihood Estimator). A random variable X has the following probability density function:

$$f(x; \theta) = \frac{1}{2\theta^3} x^2 e^{-x/\theta}, \quad 0 < x < \infty, 0 < \theta < \infty.$$

Let X_1, \dots, X_n be n i.i.d. random variables following the same distribution as X . Find the maximum likelihood estimator for θ . (15 points)

Question 4 (Maximum Likelihood Estimator). A random variable X has the following probability density function:

$$f(x; \mu, \sigma^2) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(x - \mu)^2}{2\sigma^2}\right).$$

Let X_1, \dots, X_n be n i.i.d. random variables following the same distribution as X . Find the maximum likelihood estimator for μ and σ^2 . (15 points)

Question 5 (Maximum Likelihood Estimator and Method of Moment Estimator). Let X be an exponential random variable with parameter λ . Let X_1, \dots, X_n be n i.i.d. random variables following the same distribution as X .

- 1) Find the maximum likelihood estimator for λ . (15 points)
- 2) Find the method of moment estimator for λ . (15 points)