1.

(a) The sample mean

$$\bar{X} = \frac{15.2 + 14.2 + 14.0 + 12.2 + 14.4 + 12.5 + 14.3 + 14.2 + 13.5 + 11.8 + 15.2}{11} = 13.773.$$

Since

$$\frac{\bar{X} - \mu}{\sigma/\sqrt{n}} \sim \mathcal{N}(0, 1),$$

we have

$$\mathbb{P}(-Z_{0.005} \leq \frac{\bar{X} - \mu}{\sigma/\sqrt{n}} \leq Z_{0.005}) = 0.01,$$

and a 99% confidence interval is given by

$$\bar{X} - Z_{0.005}\frac{\sigma}{\sqrt{n}} \leq \mu \leq \bar{X} + Z_{0.005}\frac{\sigma}{\sqrt{n}},$$

plug in $Z_{0.005} = 2.57$, we obtain

$$13.384 \leq \mu \leq 14.162.$$

(b) The 95% lower confidence bound is

$$\mu \geq \bar{X} - Z_{0.05}\sigma/\sqrt{n},$$

plug in $Z_{0.05} = 1.64$, we obtain

$$\mu \geq 13.526.$$

(c) Here we choose the value of n, such that

$$Z_{0.025}\frac{\sigma}{\sqrt{n}} \leq 2,$$

that is

$$n \geq \left(\frac{Z_{0.025}\sigma}{2}\right)^2 = \left(\frac{1.96\sigma}{2}\right)^2 = 0.24,$$

therefore, n is at least 1.

2.

Answer: Sample size $n = 12$. The sample mean

$$\bar{x} = \frac{\sum_{i=1}^{12} x_i}{12} = 2.08,$$

the sample variance

$$s^2 = \frac{\sum_{i=1}^{12}(x_i - \bar{x})^2}{12 - 1} = 0.0245.$$

We know that

$$\frac{(n-1)s^2}{\sigma^2} \sim \chi^2(12 - 1),$$

and

$$\mathbb{P}\left(\chi^2_{0.975,11} < \frac{(n-1)s^2}{\sigma^2} < \chi^2_{0.025,11}\right) = 0.95,$$

the 95% confidence interval for σ^2 is

$$\frac{(n-1)s^2}{\chi^2_{0.025,11}} < \sigma^2 < \frac{(n-1)s^2}{\chi^2_{0.975,11}},$$

which means that the 95% confidence interval for σ is

$$\sqrt{\frac{(n-1)s^2}{\chi^2_{0.025,11}}} < \sigma < \sqrt{\frac{(n-1)s^2}{\chi^2_{0.975,11}}}$$

plug in $\chi_{0.025,11} = 21.92$, $\chi_{0.975,11} = 3.82$, we obtain

$$0.111 < \sigma < 0.266.$$

3.

(a) We combine the unsure votes with the ones who think legalization is a good idea, thus making the vote to have only 2 options: oppose the legalization, and others. According to the sample proportion model, let $\widehat{p}$ be the sample proportion opposing the legalization.

$$\widehat{p} \sim \mathcal{N}\left(p, \frac{p(1-p)}{n}\right).$$

By using $\widehat{p}(1-\widehat{p})$ as an estimation of $p(1-p)$, since $\widehat{p} = 0.32$, $n = 1346$, we have the 95% confidence interval to be:

$$\widehat{p} - Z_{0.025}\frac{\sqrt{\widehat{p}(1-\widehat{p})}}{\sqrt{n}} \leq p \leq \widehat{p} + Z_{0.025}\frac{\sqrt{\widehat{p}(1-\widehat{p})}}{\sqrt{n}},$$

which is

$$0.295 = 0.32 - 0.025 \leq p \leq 0.32 + 0.025 = 0.345.$$

It says that with probability at least 95%, the actual percentage of people opposing the legalization is within 29.5% and 34.5%.

(b) The ± 3.5 margin says with high probability, the actual percentage of people supporting/opposing/feeling unsure about legalization of marijuana is within 3.5% of the percentage shown in this poll.