

Georgia Institute of Technology
ISyE3770 - Midterm 1 Practice

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2024/02/27

Name: _____

GT Student ID: _____

1. This midterm 1 practice contains 9 pages (including this cover page) and 9 questions. Total of points is 100.
2. In taking this examination, you are expected to adhere to the GT academic honor code. At a minimum, this requires that you utilize only the materials supplied to you and that you do not give help to, or accept help from, others.

Distribution of Marks

Question	Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	20	
Total:	100	

Multiple Choice (80 points, 10 points each)**Remark:** for each question, one and only one of four given choices (A, B, C, and D) is correct.

1. (10 points) From a group of 3 first-year students, 4 sophomores, 4 juniors, and 3 seniors, a committee of size 4 is randomly selected. Find the probability that the committee will consist of (a) 1 from each class; (b) 2 sophomores and 2 juniors; (c) only sophomores or juniors.

A. (a) $\frac{3 \cdot 4 \cdot 4 \cdot 3}{\binom{14}{4}}$, (b) $\frac{\binom{4}{2}\binom{4}{2}}{\binom{14}{4}}$, (c) $\frac{\binom{8}{4}}{\binom{14}{4}}$

B. (a) $\frac{3 \cdot 4 \cdot 4 \cdot 3}{\binom{14}{4}}$, (b) $\frac{\binom{4}{2}}{\binom{14}{4}}$, (c) $\frac{\binom{8}{4}}{\binom{14}{4}}$

C. (a) $\frac{3 \cdot 4 \cdot 4 \cdot 3}{\binom{14}{4}}$, (b) $\frac{\binom{4}{2}\binom{4}{2}}{\binom{14}{4}}$, (c) $\frac{\binom{8}{6}}{\binom{14}{4}}$

D. (a) $\frac{3 \cdot 4}{\binom{14}{4}}$, (b) $\frac{\binom{4}{2}\binom{4}{2}}{\binom{14}{4}}$, (c) $\frac{\binom{8}{4}}{\binom{14}{4}}$

1. _____

2. (10 points) The random variable X has the probability density function

$$f(x) = \begin{cases} ax + bx^2, & \text{if } 0 < x < 1, \\ 0, & \text{otherwise.} \end{cases}$$

If $\mathbb{E}[X] = 0.6$, find (a) $\mathbb{P}(X < \frac{1}{2})$ and (b) $\text{Var}(X)$.

A. (a) $\mathbb{P}(X < \frac{1}{2}) = 0.4$, (b) $\text{Var}(X) = 0.06$.

B. (a) $\mathbb{P}(X < \frac{1}{2}) = 0.35$, (b) $\text{Var}(X) = 0.06$.

C. (a) $\mathbb{P}(X < \frac{1}{2}) = 0.35$, (b) $\text{Var}(X) = 0.09$.

D. (a) $\mathbb{P}(X < \frac{1}{2}) = 0.4$, (b) $\text{Var}(X) = 0.09$.

2. _____

3. (10 points) Assume the length of a phone call in minutes follows exponential distribution with parameter $\theta = 10$. If someone arrives immediately ahead of you at a public telephone booth, find the probability that you will have to wait

(a) More than 10 minutes;

(b) Between 10 and 20 minutes.

A. (a) e , (b) $e^2 - e$.

B. (a) e^{-10} , (b) $e^{-10} - e^{-20}$.

C. (a) e , (b) $e^{-1} - e^{-2}$.

D. (a) e^{-1} , (b) $e^{-1} - e^{-2}$.

3. _____

4. (10 points) Find standard deviation of X if X has a Poisson distribution such that $2\mathbb{P}(X = 1) = \mathbb{P}(X = 2)$.
 A. 2 B. $\sqrt{2}$ C. $2\sqrt{2}$ D. 4

4. _____

5. (10 points) Assume X has moment generating function

$$M_X(t) = \exp(3t + 9t^2),$$

and Y has moment generating function

$$M_Y(t) = \frac{1}{(1 - 3t)^5}, \quad t < \frac{1}{3}.$$

Statement 1: Y has $\mathbb{E}[Y]$ of 15.

Statement 2: Y has $\text{Var}(Y)$ of 15.

Statement 3: X follows normal distribution $N(3, 9)$.

How many statements are correct?

- A. 0 B. 1 C. 2 D. 3

5. _____

6. (10 points) Suppose there are two well defined events, event A and event B . The probability that only one of them occurs is 0.3, and $P(A) + P(B) = 0.5$. What's the probability that at least one of them not occur?
 A. 0.3 B. 0.5 C. 0.6 D. 0.9

6. _____

7. (10 points) Suppose that $X \sim N(3, 25)$, find the largest d such that $P(X > d) \geq 0.9$.
 A. 1.28 B. -1.28 C. -3.40 D. -4.22

7. _____

8. (10 points) Products produced by a machine has a 3% defective rate. The first 10 inspections have been found to be free of defectives. What is the probability that the first defective will occur on the 15-th inspection?

- A. 0.97^4
 B. $0.97^5 \cdot 0.03$
 C. $1 - 0.97^4 \cdot 0.03$
 D. $0.97^4 \cdot 0.03$

8. _____

Regular Question (20 points)

9. (20 points) Suppose the joint probability density function of X and Y is given by

$$f(x, y) = cxy, \quad 1 < y < 2, 1 < x < 2.$$

- (i) Find the constant c . (10 points)
- (ii) Find the marginal probability density function of Y . (5 points)
- (iii) Find $P(X > 1.5, Y > 1.5)$. (5 points)

Discrete Distributions

Bernoulli

$$0 < p < 1$$

$$f(x) = p^x(1-p)^{1-x}, \quad x = 0, 1$$

$$M(t) = 1 - p + pe^t, \quad -\infty < t < \infty$$

$$\mu = p, \quad \sigma^2 = p(1-p)$$

Binomial

$$b(n, p)$$

$$0 < p < 1$$

$$f(x) = \frac{n!}{x!(n-x)!} p^x(1-p)^{n-x}, \quad x = 0, 1, 2, \dots, n$$

$$M(t) = (1-p + pe^t)^n, \quad -\infty < t < \infty$$

$$\mu = np, \quad \sigma^2 = np(1-p)$$

Geometric

$$0 < p < 1$$

$$f(x) = (1-p)^{x-1}p, \quad x = 1, 2, 3, \dots$$

$$M(t) = \frac{pe^t}{1 - (1-p)e^t}, \quad t < -\ln(1-p)$$

$$\mu = \frac{1}{p}, \quad \sigma^2 = \frac{1-p}{p^2}$$

Negative Binomial

$$0 < p < 1$$

$$r = 1, 2, 3, \dots$$

$$f(x) = \binom{x-1}{r-1} p^r(1-p)^{x-r}, \quad x = r, r+1, r+2, \dots$$

$$M(t) = \frac{(pe^t)^r}{[1 - (1-p)e^t]^r}, \quad t < -\ln(1-p)$$

$$\mu = r\left(\frac{1}{p}\right), \quad \sigma^2 = \frac{r(1-p)}{p^2}$$

Poisson

$$\lambda > 0$$

$$f(x) = \frac{\lambda^x e^{-\lambda}}{x!}, \quad x = 0, 1, 2, \dots$$

$$M(t) = e^{\lambda(e^t-1)}, \quad -\infty < t < \infty$$

$$\mu = \lambda, \quad \sigma^2 = \lambda$$

Uniform

$$m > 0$$

$$f(x) = \frac{1}{m}, \quad x = 1, 2, \dots, m$$

$$\mu = \frac{m+1}{2}, \quad \sigma^2 = \frac{m^2-1}{12}$$

Continuous Distributions

Beta

$$\alpha > 0$$

$$\beta > 0$$

$$f(x) = \frac{\Gamma(\alpha + \beta)}{\Gamma(\alpha)\Gamma(\beta)} x^{\alpha-1}(1-x)^{\beta-1}, \quad 0 < x < 1$$

$$\mu = \frac{\alpha}{\alpha + \beta}, \quad \sigma^2 = \frac{\alpha\beta}{(\alpha + \beta + 1)(\alpha + \beta)^2}$$

Chi-square

$$\chi^2(r)$$

$$r = 1, 2, \dots$$

$$f(x) = \frac{1}{\Gamma(r/2)2^{r/2}} x^{r/2-1} e^{-x/2}, \quad 0 < x < \infty$$

$$M(t) = \frac{1}{(1-2t)^{r/2}}, \quad t < \frac{1}{2}$$

$$\mu = r, \quad \sigma^2 = 2r$$

Exponential

$$\theta > 0$$

$$f(x) = \frac{1}{\theta} e^{-x/\theta}, \quad 0 \leq x < \infty$$

$$M(t) = \frac{1}{1-\theta t}, \quad t < \frac{1}{\theta}$$

$$\mu = \theta, \quad \sigma^2 = \theta^2$$

Gamma

$$\alpha > 0$$

$$\theta > 0$$

$$f(x) = \frac{1}{\Gamma(\alpha)\theta^\alpha} x^{\alpha-1} e^{-x/\theta}, \quad 0 < x < \infty$$

$$M(t) = \frac{1}{(1-\theta t)^\alpha}, \quad t < \frac{1}{\theta}$$

$$\mu = \alpha\theta, \quad \sigma^2 = \alpha\theta^2$$

Normal

$$N(\mu, \sigma^2)$$

$$-\infty < \mu < \infty$$

$$\sigma > 0$$

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2/2\sigma^2}, \quad -\infty < x < \infty$$

$$M(t) = e^{\mu t + \sigma^2 t^2/2}, \quad -\infty < t < \infty$$

$$E(X) = \mu, \quad \text{Var}(X) = \sigma^2$$

Uniform

$$U(a, b)$$

$$-\infty < a < b < \infty$$

$$f(x) = \frac{1}{b-a}, \quad a \leq x \leq b$$

$$M(t) = \frac{e^{tb} - e^{ta}}{t(b-a)}, \quad t \neq 0; \quad M(0) = 1$$

$$\mu = \frac{a+b}{2}, \quad \sigma^2 = \frac{(b-a)^2}{12}$$

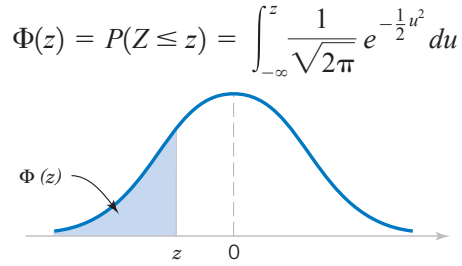


Table III Cumulative Standard Normal Distribution

<i>z</i>	-0.09	-0.08	-0.07	-0.06	-0.05	-0.04	-0.03	-0.02	-0.01	-0.00
-3.9	0.000033	0.000034	0.000036	0.000037	0.000039	0.000041	0.000042	0.000044	0.000046	0.000048
-3.8	0.000050	0.000052	0.000054	0.000057	0.000059	0.000062	0.000064	0.000067	0.000069	0.000072
-3.7	0.000075	0.000078	0.000082	0.000085	0.000088	0.000092	0.000096	0.000100	0.000104	0.000108
-3.6	0.000112	0.000117	0.000121	0.000126	0.000131	0.000136	0.000142	0.000147	0.000153	0.000159
-3.5	0.000165	0.000172	0.000179	0.000185	0.000193	0.000200	0.000208	0.000216	0.000224	0.000233
-3.4	0.000242	0.000251	0.000260	0.000270	0.000280	0.000291	0.000302	0.000313	0.000325	0.000337
-3.3	0.000350	0.000362	0.000376	0.000390	0.000404	0.000419	0.000434	0.000450	0.000467	0.000483
-3.2	0.000501	0.000519	0.000538	0.000557	0.000577	0.000598	0.000619	0.000641	0.000664	0.000687
-3.1	0.000711	0.000736	0.000762	0.000789	0.000816	0.000845	0.000874	0.000904	0.000935	0.000968
-3.0	0.001001	0.001035	0.001070	0.001107	0.001144	0.001183	0.001223	0.001264	0.001306	0.001350
-2.9	0.001395	0.001441	0.001489	0.001538	0.001589	0.001641	0.001695	0.001750	0.001807	0.001866
-2.8	0.001926	0.001988	0.002052	0.002118	0.002186	0.002256	0.002327	0.002401	0.002477	0.002555
-2.7	0.002635	0.002718	0.002803	0.002890	0.002980	0.003072	0.003167	0.003264	0.003364	0.003467
-2.6	0.003573	0.003681	0.003793	0.003907	0.004025	0.004145	0.004269	0.004396	0.004527	0.004661
-2.5	0.004799	0.004940	0.005085	0.005234	0.005386	0.005543	0.005703	0.005868	0.006037	0.006210
-2.4	0.006387	0.006569	0.006756	0.006947	0.007143	0.007344	0.007549	0.007760	0.007976	0.008198
-2.3	0.008424	0.008656	0.008894	0.009137	0.009387	0.009642	0.009903	0.010170	0.010444	0.010724
-2.2	0.011011	0.011304	0.011604	0.011911	0.012224	0.012545	0.012874	0.013209	0.013553	0.013903
-2.1	0.014262	0.014629	0.015003	0.015386	0.015778	0.016177	0.016586	0.017003	0.017429	0.017864
-2.0	0.018309	0.018763	0.019226	0.019699	0.020182	0.020675	0.021178	0.021692	0.022216	0.022750
-1.9	0.023295	0.023852	0.024419	0.024998	0.025588	0.026190	0.026803	0.027429	0.028067	0.028717
-1.8	0.029379	0.030054	0.030742	0.031443	0.032157	0.032884	0.033625	0.034379	0.035148	0.035930
-1.7	0.036727	0.037538	0.038364	0.039204	0.040059	0.040929	0.041815	0.042716	0.043633	0.044565
-1.6	0.045514	0.046479	0.047460	0.048457	0.049471	0.050503	0.051551	0.052616	0.053699	0.054799
-1.5	0.055917	0.057053	0.058208	0.059380	0.060571	0.061780	0.063008	0.064256	0.065522	0.066807
-1.4	0.068112	0.069437	0.070781	0.072145	0.073529	0.074934	0.076359	0.077804	0.079270	0.080757
-1.3	0.082264	0.083793	0.085343	0.086915	0.088508	0.090123	0.091759	0.093418	0.095098	0.096801
-1.2	0.098525	0.100273	0.102042	0.103835	0.105650	0.107488	0.109349	0.111233	0.113140	0.115070
-1.1	0.117023	0.119000	0.121001	0.123024	0.125072	0.127143	0.129238	0.131357	0.133500	0.135666
-1.0	0.137857	0.140071	0.142310	0.144572	0.146859	0.149170	0.151505	0.153864	0.156248	0.158655
-0.9	0.161087	0.163543	0.166023	0.168528	0.171056	0.173609	0.176185	0.178786	0.181411	0.184060
-0.8	0.186733	0.189430	0.192150	0.194894	0.197662	0.200454	0.203269	0.206108	0.208970	0.211855
-0.7	0.214764	0.217695	0.220650	0.223627	0.226627	0.229650	0.232695	0.235762	0.238852	0.241964
-0.6	0.245097	0.248252	0.251429	0.254627	0.257846	0.261086	0.264347	0.267629	0.270931	0.274253
-0.5	0.277595	0.280957	0.284339	0.287740	0.291160	0.294599	0.298056	0.301532	0.305026	0.308538
-0.4	0.312067	0.315614	0.319178	0.322758	0.326355	0.329969	0.333598	0.337243	0.340903	0.344578
-0.3	0.348268	0.351973	0.355691	0.359424	0.363169	0.366928	0.370700	0.374484	0.378281	0.382089
-0.2	0.385908	0.389739	0.393580	0.397432	0.401294	0.405165	0.409046	0.412936	0.416834	0.420740
-0.1	0.424655	0.428576	0.432505	0.436441	0.440382	0.444330	0.448283	0.452242	0.456205	0.460172
0.0	0.464144	0.468119	0.472097	0.476078	0.480061	0.484047	0.488033	0.492022	0.496011	0.500000

$$\Phi(z) = P(Z \leq z) = \int_{-\infty}^z \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}u^2} du$$

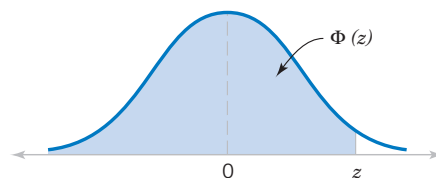


Table III Cumulative Standard Normal Distribution (continued)

<i>z</i>	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.500000	0.503989	0.507978	0.511967	0.515953	0.519939	0.523922	0.527903	0.531881	0.535856
0.1	0.539828	0.543795	0.547758	0.551717	0.555760	0.559618	0.563559	0.567495	0.571424	0.575345
0.2	0.579260	0.583166	0.587064	0.590954	0.594835	0.598706	0.602568	0.606420	0.610261	0.614092
0.3	0.617911	0.621719	0.625516	0.629300	0.633072	0.636831	0.640576	0.644309	0.648027	0.651732
0.4	0.655422	0.659097	0.662757	0.666402	0.670031	0.673645	0.677242	0.680822	0.684386	0.687933
0.5	0.691462	0.694974	0.698468	0.701944	0.705401	0.708840	0.712260	0.715661	0.719043	0.722405
0.6	0.725747	0.729069	0.732371	0.735653	0.738914	0.742154	0.745373	0.748571	0.751748	0.754903
0.7	0.758036	0.761148	0.764238	0.767305	0.770350	0.773373	0.776373	0.779350	0.782305	0.785236
0.8	0.788145	0.791030	0.793892	0.796731	0.799546	0.802338	0.805106	0.807850	0.810570	0.813267
0.9	0.815940	0.818589	0.821214	0.823815	0.826391	0.828944	0.831472	0.833977	0.836457	0.838913
1.0	0.841345	0.843752	0.846136	0.848495	0.850830	0.853141	0.855428	0.857690	0.859929	0.862143
1.1	0.864334	0.866500	0.868643	0.870762	0.872857	0.874928	0.876976	0.878999	0.881000	0.882977
1.2	0.884930	0.886860	0.888767	0.890651	0.892512	0.894350	0.896165	0.897958	0.899727	0.901475
1.3	0.903199	0.904902	0.906582	0.908241	0.909877	0.911492	0.913085	0.914657	0.916207	0.917736
1.4	0.919243	0.920730	0.922196	0.923641	0.925066	0.926471	0.927855	0.929219	0.930563	0.931888
1.5	0.933193	0.934478	0.935744	0.936992	0.938220	0.939429	0.940620	0.941792	0.942947	0.944083
1.6	0.945201	0.946301	0.947384	0.948449	0.949497	0.950529	0.951543	0.952540	0.953521	0.954486
1.7	0.955435	0.956367	0.957284	0.958185	0.959071	0.959941	0.960796	0.961636	0.962462	0.963273
1.8	0.964070	0.964852	0.965621	0.966375	0.967116	0.967843	0.968557	0.969258	0.969946	0.970621
1.9	0.971283	0.971933	0.972571	0.973197	0.973810	0.974412	0.975002	0.975581	0.976148	0.976705
2.0	0.977250	0.977784	0.978308	0.978822	0.979325	0.979818	0.980301	0.980774	0.981237	0.981691
2.1	0.982136	0.982571	0.982997	0.983414	0.983823	0.984222	0.984614	0.984997	0.985371	0.985738
2.2	0.986097	0.986447	0.986791	0.987126	0.987455	0.987776	0.988089	0.988396	0.988696	0.988989
2.3	0.989276	0.989556	0.989830	0.990097	0.990358	0.990613	0.990863	0.991106	0.991344	0.991576
2.4	0.991802	0.992024	0.992240	0.992451	0.992656	0.992857	0.993053	0.993244	0.993431	0.993613
2.5	0.993790	0.993963	0.994132	0.994297	0.994457	0.994614	0.994766	0.994915	0.995060	0.995201
2.6	0.995339	0.995473	0.995604	0.995731	0.995855	0.995975	0.996093	0.996207	0.996319	0.996427
2.7	0.996533	0.996636	0.996736	0.996833	0.996928	0.997020	0.997110	0.997197	0.997282	0.997365
2.8	0.997445	0.997523	0.997599	0.997673	0.997744	0.997814	0.997882	0.997948	0.998012	0.998074
2.9	0.998134	0.998193	0.998250	0.998305	0.998359	0.998411	0.998462	0.998511	0.998559	0.998605
3.0	0.998650	0.998694	0.998736	0.998777	0.998817	0.998856	0.998893	0.998930	0.998965	0.998999
3.1	0.999032	0.999065	0.999096	0.999126	0.999155	0.999184	0.999211	0.999238	0.999264	0.999289
3.2	0.999313	0.999336	0.999359	0.999381	0.999402	0.999423	0.999443	0.999462	0.999481	0.999499
3.3	0.999517	0.999533	0.999550	0.999566	0.999581	0.999596	0.999610	0.999624	0.999638	0.999650
3.4	0.999663	0.999675	0.999687	0.999698	0.999709	0.999720	0.999730	0.999740	0.999749	0.999758
3.5	0.999767	0.999776	0.999784	0.999792	0.999800	0.999807	0.999815	0.999821	0.999828	0.999835
3.6	0.999841	0.999847	0.999853	0.999858	0.999864	0.999869	0.999874	0.999879	0.999883	0.999888
3.7	0.999892	0.999896	0.999900	0.999904	0.999908	0.999912	0.999915	0.999918	0.999922	0.999925
3.8	0.999928	0.999931	0.999933	0.999936	0.999938	0.999941	0.999943	0.999946	0.999948	0.999950
3.9	0.999952	0.999954	0.999956	0.999958	0.999959	0.999961	0.999963	0.999964	0.999966	0.999967

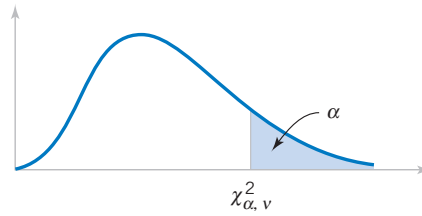


Table IV Percentage Points $\chi^2_{\alpha, \nu}$ of the Chi-Squared Distribution

$\nu \backslash \alpha$.995	.990	.975	.950	.900	.500	.100	.050	.025	.010	.005
1	.00+	.00+	.00+	.00+	.02	.45	2.71	3.84	5.02	6.63	7.88
2	.01	.02	.05	.10	.21	1.39	4.61	5.99	7.38	9.21	10.60
3	.07	.11	.22	.35	.58	2.37	6.25	7.81	9.35	11.34	12.84
4	.21	.30	.48	.71	1.06	3.36	7.78	9.49	11.14	13.28	14.86
5	.41	.55	.83	1.15	1.61	4.35	9.24	11.07	12.83	15.09	16.75
6	.68	.87	1.24	1.64	2.20	5.35	10.65	12.59	14.45	16.81	18.55
7	.99	1.24	1.69	2.17	2.83	6.35	12.02	14.07	16.01	18.48	20.28
8	1.34	1.65	2.18	2.73	3.49	7.34	13.36	15.51	17.53	20.09	21.96
9	1.73	2.09	2.70	3.33	4.17	8.34	14.68	16.92	19.02	21.67	23.59
10	2.16	2.56	3.25	3.94	4.87	9.34	15.99	18.31	20.48	23.21	25.19
11	2.60	3.05	3.82	4.57	5.58	10.34	17.28	19.68	21.92	24.72	26.76
12	3.07	3.57	4.40	5.23	6.30	11.34	18.55	21.03	23.34	26.22	28.30
13	3.57	4.11	5.01	5.89	7.04	12.34	19.81	22.36	24.74	27.69	29.82
14	4.07	4.66	5.63	6.57	7.79	13.34	21.06	23.68	26.12	29.14	31.32
15	4.60	5.23	6.27	7.26	8.55	14.34	22.31	25.00	27.49	30.58	32.80
16	5.14	5.81	6.91	7.96	9.31	15.34	23.54	26.30	28.85	32.00	34.27
17	5.70	6.41	7.56	8.67	10.09	16.34	24.77	27.59	30.19	33.41	35.72
18	6.26	7.01	8.23	9.39	10.87	17.34	25.99	28.87	31.53	34.81	37.16
19	6.84	7.63	8.91	10.12	11.65	18.34	27.20	30.14	32.85	36.19	38.58
20	7.43	8.26	9.59	10.85	12.44	19.34	28.41	31.41	34.17	37.57	40.00
21	8.03	8.90	10.28	11.59	13.24	20.34	29.62	32.67	35.48	38.93	41.40
22	8.64	9.54	10.98	12.34	14.04	21.34	30.81	33.92	36.78	40.29	42.80
23	9.26	10.20	11.69	13.09	14.85	22.34	32.01	35.17	38.08	41.64	44.18
24	9.89	10.86	12.40	13.85	15.66	23.34	33.20	36.42	39.36	42.98	45.56
25	10.52	11.52	13.12	14.61	16.47	24.34	34.28	37.65	40.65	44.31	46.93
26	11.16	12.20	13.84	15.38	17.29	25.34	35.56	38.89	41.92	45.64	48.29
27	11.81	12.88	14.57	16.15	18.11	26.34	36.74	40.11	43.19	46.96	49.65
28	12.46	13.57	15.31	16.93	18.94	27.34	37.92	41.34	44.46	48.28	50.99
29	13.12	14.26	16.05	17.71	19.77	28.34	39.09	42.56	45.72	49.59	52.34
30	13.79	14.95	16.79	18.49	20.60	29.34	40.26	43.77	46.98	50.89	53.67
40	20.71	22.16	24.43	26.51	29.05	39.34	51.81	55.76	59.34	63.69	66.77
50	27.99	29.71	32.36	34.76	37.69	49.33	63.17	67.50	71.42	76.15	79.49
60	35.53	37.48	40.48	43.19	46.46	59.33	74.40	79.08	83.30	88.38	91.95
70	43.28	45.44	48.76	51.74	55.33	69.33	85.53	90.53	95.02	100.42	104.22
80	51.17	53.54	57.15	60.39	64.28	79.33	96.58	101.88	106.63	112.33	116.32
90	59.20	61.75	65.65	69.13	73.29	89.33	107.57	113.14	118.14	124.12	128.30
100	67.33	70.06	74.22	77.93	82.36	99.33	118.50	124.34	129.56	135.81	140.17

ν = degrees of freedom.