1. Use method of enumeration

- case 1: 3 blue $\binom{7}{3}$ ways
- case 2: 2 blue + 1 red $\binom{7}{2}\binom{3}{1}$ ways.

B

2. $\int_0^1 f(x)\,dx = 1 \Rightarrow c\cdot\left[\frac{1}{2}x^2 - \frac{1}{3}x^3\right]_0^1 = 1 \Rightarrow c = 6$

$P(\frac{\sqrt{3}}{3} \le X \le \frac{5}{4}) = \int_{\frac{\sqrt{3}}{3}}^{1} f(x)\,dx = c\cdot\left[\frac{1}{2}x^2 - \frac{1}{3}x^3\right]_{\frac{\sqrt{3}}{3}}^{1}$

$= 6\cdot\left[\frac{1}{6} - \frac{1}{6} + \frac{\sqrt{3}}{27}\right] = \frac{2\sqrt{3}}{9}$

B

3. $X_i \sim \text{Poisson}(4)$, $i = 1, 2, 3, 4, 5$

P(exactly 4 of 5 received no requests)

$= \binom{5}{4}\cdot P(X_i = 0)^4 \cdot P(X_i > 0)$

$= \binom{5}{4}(e^{-4})^4(1 - e^{-4})$

C.

4. $2(1-p) = p \Rightarrow p = \frac{2}{3} \Rightarrow \sigma^2 = p(1-p) = \frac{2}{9}$

$\sigma = \frac{\sqrt{2}}{3}$.

B.

5. $X \sim Poisson(16) \Rightarrow \sigma_x^2 = 16 \quad \sigma_x = 4. \quad P(X=0) = e^{-16}$

$Y \sim \chi^2(1) \quad \Rightarrow P(Y < d) = 0.95 \Leftrightarrow d = 3.84.$

<u>C</u>.

6. $P(A \cap B) = 0.2 \Rightarrow P(A \cap B') = P(A) - P(A \cap B) = 0.2$

$P(A|B) = \dfrac{P(A \cap B)}{P(B)} = 0$ since $P(A \cap B) = 0$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$ since they're independent

$= P(A) + P(B) - P(A)P(B)$

$= 0.7.$

<u>C</u>.

7. $P(|X| \leq c) \leq 0.8064 \Leftrightarrow P(X \leq -c) = \frac{1}{2}(1 - 0.8064)$

$= 0.0968$

$\Leftrightarrow -c = -1.3000.$

<u>B</u>.

8. $X \sim exp(2) \Rightarrow F_X(x) = 1 - e^{-x/2}$

$P(X \geq 3) = 1 - F_X(3) = e^{-3/2}.$ <u>A</u>.

9.

(i). $\int_1^c \int_0^4 f(x,y)\,dx\,dy = 1 \Rightarrow \int_1^c y\,dy = \left[\frac{1}{2}y^2\right]_1^c = 1$

$\Rightarrow \frac{1}{2}(c^2-1) = 1 \qquad c = \sqrt{3}.$

(ii). $f_X(x) = \int_1^c f(x,y)\,dy = \left[\frac{1}{16}xy^2\right]_{y=1}^c = \frac{c^2-1}{16}x = \frac{x}{8}, \quad 0 \le x \le 4.$

(iii). $f_Y(y) = \int_0^4 f(x,y)\,dx = \left[\frac{1}{16}x^2y\right]_{x=0}^4 = y, \quad 1 \le y \le \sqrt{3}.$