AIE1901 Assignment 2

Due date: 11:59 PM, Wednesday, November 12, 2025.

Remark:

1) The Maximum point is 100.
2) It is okay to use LLM (such as ORLM, DeepSeek) to help you generate the answer, but it is optional.

Question 1 (Newsvendor Problem)**.** *Suppose we are running a business retailing newspapers on the CUHKSZ campus. We have to order a specific number of copies from the publisher every evening and sell them the next day. The demand is uncertain: one day it might be very high, another day it might be low. As a retailer, you need to determine the optimal number of copies to order daily to handle this demand variability.*

The key factors in your decision are:

- *Selling price (p): The revenue per newspaper sold;*
- *Buying price (c_v): The cost per newspaper purchased;*
- *Holding cost (h): The cost per unit for leftover newspapers (e.g., disposal or storage);*
- *Backorder cost (b): The cost per unit for unmet demand (e.g., loss of goodwill, lost profit);*
- *Your order quantity (y): Your decision variable, the number of newspapers to order.*

Assume the following values:

$$p = 10, c_v = 5, h = 2, b = 3.$$

Your goal is to maximize profit. The profit function is defined as revenue minus total cost:

$$\begin{aligned} \textit{Profit} &= \textit{Revenue} - \textit{Cost} \\ &= \textit{Revenue} - \textit{Ordering Cost} - \textit{Holding Cost} - \textit{Backorder Cost} \\ &= p \cdot \min(D, y) - yc_v - h \cdot \max(y - D, 0) - b \cdot \max(D - y, 0). \end{aligned}$$

The profit equation above contains a random element, the demand D. But we know that

$$\mathbf{Pr}(D = 10) = \frac{1}{4}, \quad \mathbf{Pr}(D = 15) = \frac{1}{8}, \quad \mathbf{Pr}(D = 20) = \frac{1}{8}, \quad \mathbf{Pr}(D = 25) = \frac{1}{4}, \quad \mathbf{Pr}(D = 30) = \frac{1}{4}.$$

Since the profit depends on the random demand D, we optimize for the expected profit. What is your optimal order quantity y to maximize the expected profit? *(50 points)*

Remark:

1) *Formulate this question into a mathematical optimization model and use `coptpy` or `cvxpy` package to generate answer.*
2) *Attach the computer code, prompt you to use in your submission.*

Question 2 (Supply Chain with Inventory Substitution)**.** *An airline company has* 3 *types of products to satisfy customer demands, with product* $1, 2, 3$ *corresponding to* `Business`, `Comfort`, *and* `Economy` *zones, respectively. There is a demand class corresponding to each product, indexed by* $j = 1, 2, 3$. *If any demand class* j *cannot be satisfied, products with* ***higher*** *quality can be used for substitution (However, you cannot substitute lower-quality products for higher-quality demands). Let* $y = (y_1, y_2, y_3)^\top$ *denote the inventory level of all products, and* $d = (d_1, d_2, d_3)^\top$ *denote the demands of all products. The goal is to find the optimal substitution decision. There are several factors to be considered:*

- *Substitution cost: the unit substitution cost to use product* i *to satisfy demand* j *is* s_{ij}*;*
- *Backorder cost: Unit backorder cost for demand* j *is* b_j*;*
- *Holding cost: Unit holding cost for product* i *is* h_i*;*
- *Substitution decision* ($\omega_{i,j}, \forall i, j$)*: the amount of substitution of product* i *to demand* j*, decision variable.*
- *Leftover inventory of product* i (u_i)*: decision variable, i.e., number of seats left for* i*-th product.*
- *Shortage of demand* j (u'_j)*: decision variable, i.e., number of customers who have demand* j *but fail to onboard.*

Assume the following values of our data:

$$h = \begin{pmatrix} h_1 \\ h_2 \\ h_3 \end{pmatrix} = \begin{pmatrix} 20 \\ 15 \\ 10 \end{pmatrix}, \qquad b = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} = \begin{pmatrix} 500 \\ 300 \\ 280 \end{pmatrix}$$

$$s = \begin{pmatrix} s_{11} & s_{12} & s_{13} \\ s_{21} & s_{22} & s_{23} \\ s_{31} & s_{32} & s_{33} \end{pmatrix} = \begin{pmatrix} 0 & 50.6 & 47.6 \\ \infty & 0 & 140.7 \\ \infty & \infty & 0 \end{pmatrix}$$

Suppose now the inventory level y *and* d *are given:*

$$y = \begin{pmatrix} 100 \\ 200 \\ 300 \end{pmatrix}, \quad d = \begin{pmatrix} 72 \\ 157 \\ 326 \end{pmatrix}.$$

What is your optimal decision for this question? *(50 points)*

Remark:

1) *Formulate this question into a mathematical optimization model and use* `coptpy` *or* `cvxpy` *package to generate answer.*
2) *Attach the computer code, prompt you to use in your submission.*
3) *For your reading interest, this problem is a simplified version of the practical question published in Operations Research (see* `Xin Chen, Xiangyu Gao (2019) Stochastic Optimization with Decisions Truncated by Positively Dependent Random Variables. Operations Research 67(5): 1321-1327. https://doi.org/10.1287/opre.2018.1815`*).*