Desert Survival Optimization Problem

A Mixed-Integer Programming Approach

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Outline

A quick start

- "Oxygen Not Included" is a space colony simulation game. The player is required to establish a colony to survive on a new planet.
- Can optimization play an important role here? Is there any "optimal" strategy for the player to survive?

A quick start

Colony Management in "Oxygen Not Included"

1. Parameters (Fixed)

Known constants in the problem:

- Duplicant O consumption: 100g/s
- Electrolyzer efficiency: 1kg $H_2O \rightarrow$ 888g O_2
- Coal generator output: 600W

2. Decision Variables

What we control and optimize:

- # of electrolyzers: x_1
- # of coal generators: x_2
- Food production rate: x_3

3. Constraints (Limits)

Rules we must follow:

- Power: $600x_2 \geq$ Total consumption
- Oxygen: $888x_1 \geq 100n$ duplicants
- Water: Available $1000x_1$
- Space & heat limitations

4. Objective Function

What we want to optimize:

$$\text{Max: Colony survival} \tag{1}$$

$$\text{Min: Resource waste} \tag{2}$$

Game Overview

Desert Survival Game:
A player traverses a desert map using initial capital to purchase water and food supplies. The goal is to reach the destination within a specified time while maximizing remaining capital.

- **Map**: Hexagonal grid with numbered regions
- **Starting point**: Region 1
- **Destination**: Region 64 (must reach by deadline)
- **Resources**: Water and food with daily consumption
- **Weather**: Affects resource consumption (Clear, Hot, Sandstorm)
- **Activities**: Travel, stay, mine for gold, purchase supplies

Map Structure

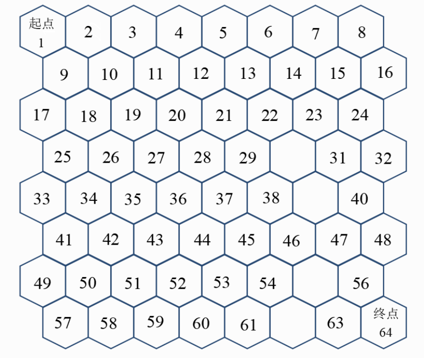

Figure: Map

- Adjacent regions share boundaries or vertices
- Travel between adjacent regions only
- Some regions contain mines for gold extraction

Parameter Settings

Game Parameters:

- **Weight limit**: 1200 kg
- **Initial capital**: 10,000 yuan
- **Base mining income**: 1000 yuan
- **Deadline**: Day 30

Resource Information:

Resource	Weight per Box (kg)	Base Price (yuan/box)
Water	3	5
Food	2	10

Weather Forecast

Day	1	2	3	4	5	6	7	8	9	10
Weather	Hot	Hot	Clear	Sand	Clear	Hot	Sand	Clear	Hot	Hot
Day	11	12	13	14	15	16	17	18	19	20
Weather	Sand	Hot	Clear	Hot	Hot	Hot	Sand	Sand	Hot	Hot
Day	21	22	23	24	25	26	27	28	29	30
Weather	Clear	Clear	Hot	Clear	Sand	Hot	Clear	Clear	Hot	Hot

Sandstorm days: 4, 7, 11, 17, 18, 25 (player must stay in place)

Basic Game Rules

1. **Time units**: Days (game starts on Day 0)
2. **Deadline**: Must reach destination by or before the deadline
3. **Resource consumption**: Water and food consumed daily
4. **Weather conditions**: Clear, Hot, Sandstorm (affects entire map)
5. **Movement**: Can move to adjacent regions or stay in current region
6. **Resource management**:
   - Base consumption when staying $= 2\times$ base consumption when moving
   - Sandstorms require staying in place

Resource Consumption Rules

Daily Consumption:

- **Base consumption**: Minimum daily requirement
- **Staying**: Consumes base amount
- **Moving**: Consumes $2\times$ base amount
- **Mining**: Consumes $3\times$ base amount (if mining capability exists)

Weather Effects on Base Consumption:

Weather	Water	Food
Clear	5	7
Hot	8	6
Sandstorm	10	10

Trading and Mining Rules

Resource Trading:

- Initial purchase at starting location with base prices
- Can return to starting location to purchase more (not multiple times per day)
- Leftover resources can be sold back at 50% of base price when reaching destination

Mining Operations:

- Available in designated mine regions
- Mining income = base income (1000 units)
- Can choose to mine or not mine each day at mine locations
- Cannot move forward on sandstorm days

Objective Function

Goal: Maximize final capital: remaining money + value of leftover resources

$$\max \quad s_T + 0.5 \sum_{k=1}^{K} v_{T,k} P_k \tag{3}$$

Where:

- s_T: Remaining money at deadline
- $v_{T,k}$: Quantity of resource type k at deadline
- P_k: Base price of resource type k
- 0.5: Sell-back rate (50% of base price)

Decision Variables

Key Decision Variables:

- $x_{t,i}$: Binary variable indicating if player is at region i on day t
- $y_{t,i}$: Binary variable indicating if player stays at region i on day t
- $z_{t,k}$: Integer variable for quantity of resource k purchased on day t
- w_t: Binary variable indicating if player mines on day t
- $u_{t,k}$: Quantity of resource k before purchase on day t
- $v_{t,k}$: Quantity of resource k after purchase on day t
- s_t: Amount of money left on day t (after consumption)
- $a_{t,k}$: arrive at k resource place on day t
- b_t: whether it is available to dig mine on day t
- g_t: the benefit on day t

Define Variables

1. **Resource Point**: $a_{t,k} = \sum_{i=1}^{N} x_{i,t} D_{i,k}$
2. **Mine point**: $b_t = \sum_{i=1}^{N} y_{t,i} E_i$
3. **Arrive at destination**: $d_t = \sum_{i=1}^{N} x_{t,i} F_i$
4. **Mine benefit**: $g_t = \sum_{i=1}^{N} y_{t,i} E_i G_i$

Where:

- $D_{i,k}$: a binary variable that indicates whether location i can purchase resource k
- E_i: a binary variable represents whether location i have mine
- F_i: a binary variable indicates whether location i is destination
- G_i: the potential benefit if player dig mine at location i

Resource State Transition

Resource Evolution:
Resources before purchase on day t:

$$u_{t,k} = v_{t-1,k} - \left(2w_t - \sum_{i=1}^{N} y_{t,i} + 2 - d_{t-1}\right) A_{t,k} \tag{4}$$

Where:

- $A_{t,k}$: Base consumption of resource k on day t (weather-dependent)
- The term $(2w_t - \sum_{i=1}^{N} y_{t,i} + 2 - d_{t-1})$ determines consumption multiplier:
  - If staying: consumes $2 \times A_{t,k}$
  - If moving: consumes $A_{t,k}$
  - If mining: consumes $3 \times A_{t,k}$

Money Dynamics

Capital Evolution:

Initial capital after purchasing resources:

$$s_0 = J - \sum_{k=1}^{K} z_{0,k} P_k \tag{5}$$

Daily capital update:

$$s_t = s_{t-1} + g_t w_t - \sum_{k=1}^{K} z_{t,k} P_k \tag{6}$$

Where:

- J: Initial capital (10,000 yuan)
- g_t: Mining income on day t (1,000 yuan if mining)
- P_k: Base price of resource k

Movement Constraints

Location and Movement Rules:

1. **Single location**: $\sum_{i=1}^{N} x_{t,i} = 1, \quad \forall t$
2. **Adjacent movement**: $x_{t,i} + x_{t-1,j} \leq H_{i,j} + 1$
3. **Sandstorm restriction**: $\sum_{i=1}^{N} y_{t,i} = 1, \quad \forall t \in B$ (sandstorm days)
4. **Starting position**: $x_{0,1} = 1$
5. **Must reach destination**: $x_{T,N} = 1$

Stay at destination : $d_t \leq d_{t-1}$
Where $H_{i,j} = 1$ if regions i and j are adjacent, 0 otherwise.

Resource and Mining Constraints

Resource Management:

1. **Survival condition**: $u_{t,k} \geq 0$ (cannot run out of resources)
2. **Weight limit**: $\sum_{k=1}^{K} v_{t,k} W_k \leq L$ (carrying capacity)
3. **Mining capability**: $w_t \leq b_t$ (can mine only when capable)
4. **Resource purchase**: $z_{t,k} \leq M a_{t,k}$ (Noted: M is a sufficiently large number; can buy only at resource points)
5. **Resource transition**: $v_{t,k} = u_{t,k} + z_{t,k}$

Where:

- W_k: Weight per unit of resource k
- L: Weight carrying limit (1200 kg)
- b_t: Mining capability on day t
- $a_{t,k}$: Availability of resource k for purchase on day t

Logical Constraints

Stay/Move Logic:
The relationship between location and staying:

$$y_{t,i} = x_{t-1,i} \cdot x_{t,i} \tag{7}$$

Linearized as:

$$y_{t,i} \leq x_{t,i} \tag{8}$$
$$y_{t,i} \leq x_{t-1,i} \tag{9}$$
$$x_{t-1,i} + x_{t,i} \leq y_{t,i} + 1 \tag{10}$$

This ensures $y_{t,i} = 1$ if and only if the player is at region i on both days $t-1$ and t.

Model Characteristics

Problem Type: This is a **Mixed-Integer Linear Programming (MILP)** problem with:

- **Binary variables**: Location, staying, mining decisions
- **Integer variables**: Resource purchase quantities
- **Continuous variables**: Resource levels, money
- **Linear constraints**: All constraints are linear
- **Linear objective**: Maximize final wealth

Solution Methods:

- Commercial solvers: Gurobi, CPLEX, XPRESS, COPT
- Open-source solvers: SCIP, CBC
- Modeling languages: AMPL, GAMS, PuLP (Python)

Expected Solution Strategy

Optimal Strategy Components:

1. **Resource Planning**: Buy sufficient supplies initially, considering weather patterns
2. **Path Selection**: Choose route that balances travel time with mining opportunities
3. **Mining Decisions**: Mine when profitable and time permits
4. **Weather Adaptation**: Adjust consumption planning for known weather conditions
5. **Risk Management**: Maintain safety margins for unexpected delays

Key Trade-offs:

- Speed vs. mining income
- Resource safety vs. carrying capacity
- Direct path vs. mining detours

Model Extensions

Possible Enhancements:

1. **Uncertainty**: Stochastic weather or resource prices
2. **Multi-objective**: Balance profit, risk, and completion time
3. **Dynamic trading**: Resource prices vary by location and time
4. **Equipment**: Tools that reduce consumption or enable new activities
5. **Multiple players**: Game-theoretic competition

Real-world Applications:

- Supply chain optimization
- Military logistics planning
- Emergency evacuation planning
- Resource exploration missions

Thank You!

Questions and Discussion